Technical Publications

Capacitive Energy Conversion

Energy Theory

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4/3/24	Further analysis of yield in section 5
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19/4/22	Addition of energy profiling and torque calculation
15/4/22	Removal of "Experimental Validation" *et seq* for inclusion in a separate report Revised title accordingly
15/5/21	Various corrections in "Work and Energy"
9/5/21	Corrections to "A More Efficient Cycle"
19/4/21	Revision of "Reality Check" for comparison with PV panels
18/4/21	Clarifications in the "Energy Recovery" section
17/4/21	New section added "Energy Recovery". Other minor corrections
9/4/21	Subsection added " A second experiment"
7/4/21	New section added "Experimental Validation". Other minor corrections
6/4/21	Spelling correction front page
5/4/21	New section added "A More Efficient Cycle"
5/4/21	Spelling correction P3
30/3/21	New sections added "Background" and "Charge and Discharge"
27/3/21	Initial report

SUMMARY

This report explores the theory and mechanical implications of using variations in electrical capacitance to collect mechanical energy and render it available in electrical form. Fully developed, it may represent a feasible alternative to the common dynamo or alternator, applicable to energy sources other than rotation (such as variations in pressure), and may be scalable to any required capacity.

BACKGROUND

The generally accepted name for this type of theory/device is Electrostatic Transducers (ETs). ETs fall into two main types: those that convert electrical signals into mechanical energy, and those that do the opposite.

The former have been the subject of product development for many years, and have spawned two principal types of commercial devices: one is the widely-known variety of Hi Fi equipment known as electrostatic speakers, and the other is ultrasound devices used for example in cleaning baths and sub-surface communication. But neither of these are found to have any useful input to the problem at hand.

However the latter have spawned a few academic papers since the early 2000s. To this end, continuing efforts are being invested in discovering different forms of mechanically variable capacitor, but so far they have all come up against two problems. The first is that they are mechanically inefficient, which means that there are significant mechanical energy losses (e.g. in moving fluids in tight spaces, or bending or stretching materials) in addition to the theoretical requirement for the electrical output, so they have a low yield. The other is that they are not seen as scalable, which limits them to micro-applications such as instrumentation and portable applications. The result is that for the time being they remain largely a laboratory curiosity rather than a clear commercial reality. The author's access to academic papers and journals ceased when he completed his PhD in 2007, which makes it difficult to keep up. But if anybody had solved the above problems it is certain that we would all soon have known about it.

Having said that, when the author worked on lasers in the late '60s, they were considered as no more than a laboratory curiosity or "a solution in search of a problem", and it was several years before lasers of sufficient power and stability found favour as industrial tools. So, in this case, the door remains open and while the author does not have access to exotic materials or resources for Micro Electro-Mechanical Systems (MEMS), there are yet likely to be interesting opportunities for discovery.

BASIC THEORY

When two conducting plates are placed close together and an electrical charge is transferred between them, then an electrical field is established, and an electrostatic force of attraction exists between the two plates. The force depends on the amount of charge, the overlapping or common area of the plates and the permittivity of the intervening dielectric medium.

In the common use of capacitors the force is considered insignificant for several reasons including: the dielectric is usually solid and (for practical purposes) incompressible; there is no reason to want to vary the dielectric permittivity or the area of, or distance between, the plates; and the electrostatic force is not nearly high enough to cause mechanical stress problems.

For flat plates the attractive force is governed by the equation:

1.1 $F=q^2/2eA$

Where:

F is the Force in Newtons. Note that this is attractive and will discourage attempts to separate further.
q is the nett charge transferred from one plate to the other in Coulombs.
A is the common (overlapping) area of the plates in square metres.
e is the permittivity of the dielectric. That of free space (or air) is close to $8.85*10^{-12}$ Farads/metre.

The charge is the integral of current in Amps and time, from a discharged state, and the force does not depend on the separation, so long as the separation is small compared to the dimensions of the plates.

This force is not affected by the thickness of the plates as all the charge on each plate is gathered at the surface nearest the opposing charge; and if the plates are of different sizes or shapes the effective area is only where the charges gather, which is where they have a common area.

As a matter of interest, cylindrical capacitors (electrolytics) are traditionally constructed by anodising very thin and very long ribbons of aluminium and winding them up tightly. The anodising is both hard and very thin, and has a high electrical resistance, so the result is robust and can cheaply achieve much higher capacitance for a given bulk than flat-plate style components. They do however have significant disadvantages, including large variability in value (usually about 20%, although this used to be a lot higher) and high internal inductance arising from the coiled construction. These are significant limitations in high-frequency circuits, but as far as electrostatic theory is concerned they are essentially the same as flat-plate types.

The electrical "value" of a capacitor will generally depend only on the common area, the intervening distance and the permittivity of the dielectric medium, and expressed by the equation:

1.2 $C=eA/d$

Where:

C is the capacitance in Farads.
e is the permittivity of the dielectric. That of free space (or air) is close to $8.85*10^{-12}$ Farads/metre.
d is the separation in metres.

Varying either the permittivity or the separation is not usually convenient, so variable capacitors, other than solid state devices, traditionally comprise of interlaced conducting plates in an air medium, and the plates are rotated manually or by a small motor to adjust the common areas and thus the capacitance. The mechanical work involved is normally both extremely small and confined almost entirely to mechanical losses rather than electrostatic effects.

Therefore, while electrostatic forces exist in all capacitors that carry a charge, the significance, even for variable capacitors, of the electrostatic forces is not normally considered worthy of attention. However, if there is a force between charged plates, however small, then varying either the common area or the distance between the plates will necessarily involve some exchange of work and energy, and we will continue by exploring the possibilities that this presents.

WORK AND ENERGY

Work and energy, in whatever form, are the same thing with different names. And, while energy may be converted from one form to another, it is never lost or gained. Thus:

- Coal and petrol, combined with oxygen, release chemical energy as heat.
- Heat creates hot gas (or steam) which can be converted into mechanical energy.
- Mechanical energy and electrical energy can be interchanged (albeit usually with some lost as heat in the process).
- Sunlight (electromagnetic radiation) creates heat in the earth and atmosphere.
- Heat causes radiation which is transferred to remote or surrounding objects.

The apparent exception, the conversion of matter into energy according to Albert Einstein's discovery, is not really an exception as quantum mechanics has shown that matter can be seen as another form of stored energy. But wherever one form of energy is converted to another there are usually losses, particularly those arising from non-reversible thermodynamic processes. But for present purposes we need only to consider the usual mechanical suspects, such as friction, viscous and possibly aerodynamic losses, when the time comes to consider the apparatus of energy conversion.

Getting back to our electrostatic plates and equation 1.1, Newton's laws state that if you push against some resistance, then the amount of work (or energy) you put into the system is simply given by the equation:

2.1 $W=F(d_1-d_0)$

Where:

W is the work done (energy supplied) in Newton-metres.
F is the counteracting force in Newtons.
d_0 is the initial position in metres.

d_1 is the eventual position in metres.

Both positions are measured from the same arbitrary point against the direction of the counteracting force. Thus it makes no difference whether we consider d_0 to be zero and take d_1 to be the length of movement, or we take both to be the actual distances between the plates of our capacitor.

But from 1.1 the force does not vary with distance, so the work done in increasing the separation between the plates is simply found from 2.1:

2.2 $W= q^2(d_1-d_0)/2eA$

This does not take into account any kinetic energy gained or lost by virtue of the speed of the moving object(s) or any mechanical losses from squeezing or stretching the dielectric or any other part of the apparatus or operator. These things may, or may not, become significant when the time comes to consider the apparatus of energy conversion.

Now we need to consider the energy represented by the charge in the capacitor. If the capacitor is disconnected then the charge on the plates cannot change, but that doesn't mean that the energy remains the same.

The energy represented by the charge in a capacitor is given by the simple equation:

2.3 $E=qV/2$

Where:

E is the energy, in Joules.
q is the nett charge transferred from one plate to the other in Coulombs. As in 1.1.
V is the voltage between the plates.

As work or energy can be found in many forms, it can be described in a number of equivalent units. In this case, Joules (in 2.3) are exactly the same as Newton-metres (in 2.1).

The voltage on a capacitor is found from the charge and the capacitance. So:

2.4 $V=q/C$

Where:

V is the same as in 2.3
q and C are the same as in 1.1 and 1.2

So the energy can be restated by combining 2.3 and 2.4:

2.5 $E=q^2/2C$

According to these equations it makes no difference how the charge is introduced (e.g. suddenly or slowly over time). It only matters how much of it there is at the moment of interest. Similarly, if the capacitance is changed without changing the charge, it only matters how big it is at the moment of interest.

If we combine 2.5 with 1.2 we can express the energy in a capacitor in the same terms as the work done in changing it mechanically. Thus

2.6 $E=q^2d/2eA$

Where:

d, e and A are the same as in 1.2.

So if we start with a separation of d_0 and increase it to d_1, the gain in energy is simply the difference between the two states:

2.7 $E_1-E_0=q^2d_1/2eA-q^2d_0/2eA$

Where:

E_1 is the final energy
E_0 is the initial energy

and, by rearranging to find the additional electrical energy:

2.8 $E_1 - E_0 = q^2(d_1 - d_0)/2eA$

Which exactly matches the work done by the mechanical input (c.f. 2.2). So we have discovered that mechanical work done in reducing the capacitance of a charged capacitor (if d_1 is greater than d_0 then the capacitance is reduced) exactly matches the electrical energy added to the capacitor.

But the charge hasn't changed, so where is this energy?

From equation 2.3 we have the voltage on the capacitor. So $V_0 = q/C_0$ and $V_1 = q/C_1$ where q is the same throughout, and thus:

2.9 $V_1 - V_0 = q(1/C_1 - 1/C_0)$

Where:

C_1 is the final capacitance
C_0 is the initial capacitance

Which tells us that by changing the capacitance, even without changing the charge, we have changed the voltage, which is a clue. This may be a significant effect when designing the apparatus.

Now going back to 2.5 for a moment, we can express the energy transfer in terms of the fixed charge and the change in capacitance. Thus

2.10 $E = q^2/2C_1 - q^2/2C_0$

Which can be rearranged more meaningfully:

2.11 $E = q^2(C_0 - C_1)/2C_0C_1$

This suggests that to transfer the greatest energy we should aim to maximise the charge and the change in capacitance, and minimise the initial capacitance. But this has a practical implication: to put a large charge onto a small capacitor requires a large voltage (see 2.4), and this will present limitations according to the materials used.

So much for what happens if we change the capacitance by increasing the separation, but we could also do this by changing the common area. This might for example take the form of the tuning capacitors used in analogue radios, where pivoted plates are interlaced with fixed plates at constant separation.

While the areas of the plates may not have changed, the electrostatic charges will collect on the overlapping areas, so the effective capacitance will be determined predominantly by the overlapping areas rather than the plates as a whole. If the separation is small we can for practical purposes disregard edge effects.

Revisiting equation 1.2 and calculating the change in capacitance between two different area values we have:

2.12 $C_0 - C_1 = eA_0/d – eA_1/d = e(A_0 - A_1)/d$

While the overlapping areas may not be a simple function of the linear or angular displacement, depending on the shapes involved, we can now calculate the energy transfer using equation 2.11 exactly as before as it depends only on the capacitance and not on the construction or dimensions.

Revisiting equation 2.6 we can calculate the change in electrostatic energy with the change in area:

2.13 $E_1 - E_0 = q^2d/2eA_1 - q^2d/2eA_0 = q^2d(1/A_1 - 1/A_0)/2e$

Similarly by revisiting 1.1 we can calculate the change in electrostatic force with change in area:

2.14 $F_1 - F_0 = q^2/2eA_1 - q^{22}/eA_0 = q^2(1/A_1 - 1/A_0)/2e$

Rephrasing 2.1 the work done is the integral of the product of force and distance. While this is simply calculated as a product for changing the separation, it also works by changing the force. So from 2.14 we get:

2.15 $W = q^2 d(1/A_1 - 1/A_0)/2e$

Again we have shown that the work done in mechanically changing the overlapping plate area is exactly the same as the gain in electrostatic energy. The force required to change the overlap of the plates depends on the shape of the plates – displacing a pair of discs will require a very different force profile than displacing rectangles, but it can be worked out from 2.15 and the relationship between the change of area with the displacement.

CHARGE AND DISCHARGE

When charging and discharging a capacitor the voltage is continually changing. So the instantaneous power transfer is given by:

3.1 $p = iv$

Where
p is the power
i is the current
v is the voltage
All at any particular moment in time

But current is the rate of transfer of charge, so using calculus notation we also have:

3.2 $i = \delta q/\delta t$

But from 2.4, and assuming C is constant:

3.3 $\delta q = C\delta v$

So:

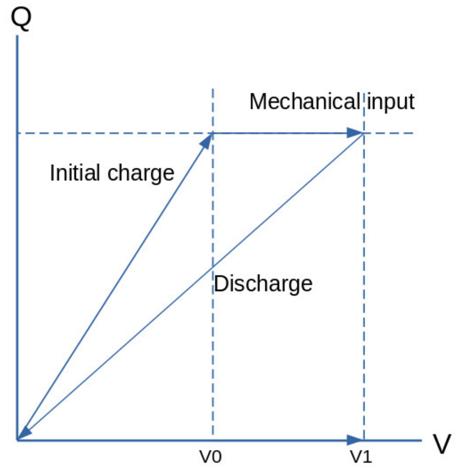
3.4 $i = C\delta v/\delta t$

And from 3.1:

3.5 $p = CV\delta v/\delta t$

Which is the rate of transfer of energy, or $\delta e/\delta t$
So the total energy E is the integral of δe, which is:

3.6 $CV^2/2$, or $QV/2$



Charge vs Voltage (Energy) Cycle

By using calculus instead of averages we make no assumptions about the rate of charge/discharge, or how it might vary during the process. So the result is independent of how it got there.

The diagram showing Q and V represents an energy diagram. It is the equivalent of the P-V diagram in gas thermodynamics, and the amount of energy transfer is represented by the area inside the inverted triangle. The area of a triangle is half the base times the height, so we find the net transfer of mechanical energy in a single cycle is given by:

3.7 $E_1 - E_0 = Q\,(V_1 - V_0)/2$

But this achieved by receiving a top-up energy for each cycle

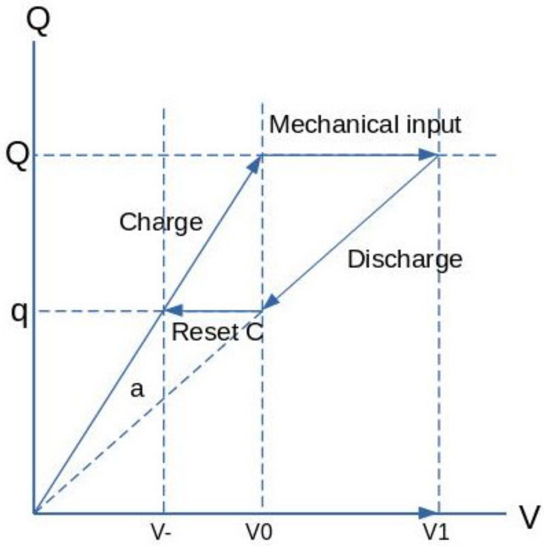
3.8 $E_0 = Q\,V_0/2$

And delivering a total electrostatic energy in each cycle

3.9 $E_1 = Q\,V_1/2$

AN ALTERNATIVE CYCLE

When charging and discharging the capacitor there will necessarily be a loss of power, however small, perhaps due to resistive losses especially when discharging at high current. So if the capacitor is only discharged as far as V_0 instead of completely, some losses can be averted and the cycle time can be shortened.



Revised Energy Cycle 1

The effect is shown in the Revised Energy Cycle 1 diagram. The transfer of energy per cycle is given by the arrowed parallelogram. There would need to be a “starter” power source to kick off the first cycle with the initial charge as before but the discharge and recharge will be less, and quicker.

Clearly the area of the parallelogram is less than the original triangle, and this will be reflected in the output energy and the mechanical demand. The discharge slope is determined solely by the capacitance, which remains low for this part of the cycle, until it is terminated at V_0.

The lower horizontal path shows the effect of increasing the capacitance while keeping the same charge (q). From the geometry the “loss” in energy transfer is given by the area of the small triangle marked “a”.

Note that this “loss” is not a loss or waste of energy but a loss of transferred energy, resulting in a reduced mechanical load.

So the energy taken from the mechanical input is the same as before

4.1 $E = Q(V_1-V_0)/2$

But some of that input is returned when the capacitance is increased

4.2 $E = q(V_0-V_-)/2$

But V_- is found from the division of triangles

4.3 $V_- = qV_0\,Q = V_0^2 V_1$

So the net mechanical input is

4.4 $E = Q(V_1-V_0)/2 - q(V_0-qV_0/Q)/2$

For best efficiency it is expected that V_1 will be several (perhaps many) times V_0, and Q will be substantially greater than q. So the compromise in loss of transferred energy by discharging only as far as V_0 should be small overall. For clarity 4.4 can be rearranged, and substituting for Q from 2.4:

4.5 $E = V_0^2C_0(V_1/V_0 - (V_0/V_1)(1-V_0/V_1) - 1)/2$

This represents the electrostatic energy harvested for each cycle, and the mechanical input, in terms of the parameters controllable by design, i.e. geometry, battery voltage and trigger voltage. It is assumed that the change in capacitance is sufficient for the voltage to reach V_1. Equation 2.4 indicates how the voltage changes with capacitance while the charge remains the same, from which the maximum trigger voltage can be determined from the available change in capacitance..

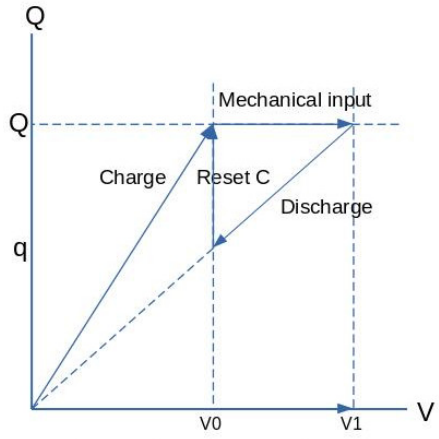
However the design must allow for the charge to be recovered from an electrical source

4.6 $E = qV_0/2$

This cycle could be implemented by having two threshold sensors on the output to the control unit: one switches the output to a charge accumulator or load when the voltage rises to a pre-determined V_1, and the other stops it when the voltage drops to V_0.

ANOTHER ALTERNATIVE CYCLE

If the minimum voltage is stabilised at V_0. For example by using a battery and diode, a different energy cycle results, as shown in Revised Energy Cycle 2 diagram. This eliminates the part of the cycle where the capacitance increases at constant charge, and the complications that arise when mechanical energy is returned.



Revised Energy Cycle 2

The resulting discharge energy for each cycle, shown as the smaller triangle top right in the energy diagram, and can be calculated

5.1 $E=(V_1 - V_0)*(Q - q)/2$

If this is compared with the energy of the initial charge

5.2 $E_0=V_0Q/2$

The net yield per cycle can be calculated by comparing these. Using simple geometry of congruent triangles this becomes

5.3 $E=E_0(V_1/V_0 + V_0/V_1 - 2)$

5.4 Since $V_1/V_0 = C_0/C_1$ this can be shown in terms of the physical geometry.

5.5 $E=E_0(C_0/C_1 + C_1/C_0 – 2)$

In conjunction with 5.2 this provides a practical and convenient way of calculating the energy yield per cycle.

The net power in Watts is important for practical applications, and this also depends on the cycle rate per second. So from 1.2, 2.4, 5.2 and 5.5, the net power yield can be found.

5.6 $P=V^2\varepsilon_0\varepsilon_r A(Cr + 1/Cr - 2)R/2d$

Where

P is the power in Watts
R is the cycle rate per second
Cr is the capacitance ratio C_0/C_1
A is the plate area
d is the plate separation
V is the supply/battery voltage
ε_0 is the permittivity of free space
ε_r is the dielectric relative permittivity

VOLTAGE AND ENERGY RATES

When considering the design characteristics of the equipment it is of interest to be aware of the rate of change of energy during the cycle; in particular how it varies with the capacitance. From this the mechanical reaction (e.g. torque in a rotating machine) can be calculated from it’s position and speed.

From 2.4 the voltage is determined by the charge and the capacitance.

6.1 $V = q/C$

Where

q is the instantaneous charge at any time
C is the instantaneous capacitance at any time

Both of these change at various points in the cycle, so the instantaneous rate of change of voltage is calculated by differentiating this. It cannot be assumed in the general case that either the rate of change of capacitance or voltage is linear in time, so it is not appropriate to simply assume a linear relationship between known limits.

Thus, at any moment in the cycle

6.2 $\partial V/\partial t = \partial/\partial t[q/C]$

This is where it might get complicated as, depending on the equipment design, the charge and capacitance may change at the same time. However, as a general approximation, we can get some useful estimate by assuming they change predominantly at different times.

So, while the capacitance is decreasing and energy is being drawn from the environment this simplifies to

6.3 $\partial V/\partial t = q.\partial/\partial t[1/C]$

It is easier to calculate the rate of change of C from the mechanical design, rather than it's inverse, but we can expand and differentiate to get what we want. So

6.4 $\partial V/\partial t = q.\partial/\partial C[1/C].\partial C/\partial t$

Which becomes, using standard differential calculus:

6.5 $\partial V/\partial t = -(q/C^2).\partial C/\partial t$

Thus we can calculate the rate of change of voltage if we know the charge and capacitance at that point, and the rate of change of capacitance with time. The negative sign is important as it tells us that a decrease in capacitance is what gives us an increase in voltage.

The rate of change of energy is then calculated from 2.3:

6.6 $E = qV/2$

So at any instant the rate of change of energy can be calculated from the known variables.

6.7 $\partial E/\partial t = -(q^2/2C^2).\partial C/\partial t$

TORQUE

In the particular case of a rotating machine, the corresponding torque can be calculated.

7.1 $T = rxF$

Where
T is the torque in metre-Newtons
r is the radius at which force is applied to the circumference, in metres
x is the cross-product operator
F is the circumferential force in Newtons

But work or energy, according to Newton, is force times distance, so

7.2 $E = Fd$

Where
F is the force
d is the distance moved

So, as the distance moved in a circle is the product of radius and angular change, and if the force does not change:

7.3 $E = Fr\Theta$

Where
E is the energy or work
F is the force in Newtons
Θ is the change in angle in radians
t is the elapsed time

But, since the force might change with the angle, differential calculus can be used to find the instantaneous change in energy at any time, so substituting:

7.4 $\partial E/\partial t = Fr\partial\Theta/\partial t$

But the torque is always the product of radius and force (as in 7.1), and $\partial\Theta/\partial t$ is the angular velocity ω, so substituting from 7.2 and losing the cross-product operator to find a scalar quantity:

7.5 $\partial E/\partial t = T\omega$

And from 6.7 we have the instantaneous torque in terms of the known variables:

7.6 $T = -\omega(q^2/2C^2).\partial C/\partial t$

Note that C and $\partial C/\partial t$ will be related to Θ and ω according to the effect of the rotation at any angular position and the device geometry. The negative sign indicates that the torque reacts against the motion that causes a decrease in the capacitance.

ADAPTABILITY

Wind turbines need to make the best of widely varying wind speeds, harvesting what they can from light breezes without being destroyed by strong winds. Similar issues arise with water turbines except where they are placed in a flow with a constant “head”. In the same way, this device may be used where there is not a regular and reliable mechanical input. Therefore a device depending on these effects must be adaptable, and this can be achieved in either of two ways.

If a low upper threshold V_1 is set by the discharge trigger, then something can be taken from small amounts of energy, although this would compromise the amount of electricity harvested when higher energies are available. A higher threshold would optimise the returns from greater energies, but would fail to trigger at low levels.

The required effect could be achieved by adjusting the initial voltage V_0, which would affect the initial charge and shift the operating voltage range; this would be a way of allowing a constant trigger threshold V_1 in response to varying source power. This would need some sort of controlling sensor. The device would also have an irregular output voltage, which may may lead to inefficiency in the load applications.

Alternatively, instead of having a fixed trigger voltage V_1, a differentiating sensor (essentially a capacitor in series with a resistor) could determine the moment at which the generated voltage is at it’s peak, and trigger at that moment whatever the peak voltage may be. This would permit optimum energy harvesting with a constant supply voltage V_0, which would be both efficient and convenient.

The above assumes that the capacitive effects may vary according to the power (e.g. amplitude) of the mechanical input. But where the architecture ensures the capacitance changes are consistently repetitive then the threshold could be set accordingly and the yield would be determined only by the rate of repetition.

In any case, while the mechanical “load” is determined only by the energy that is harvested, it remains important to take best advantage of the available energies under different conditions.

